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This is one of two presentations about digitalisation of basic metrological concepts.

Digitalisation presents us with a choice:

- Replicate what we do now, using paper-based information handling, or
- Consider how best to implement our metrological principles

In this talk we invite you to consider the second choice. Look at what we are doing today, with systems designed for people, and ask:

- What is this process trying to achieve?
- Could a digital system achieve this in a different way?

In this talk Mark and I suggest that the treatment of quantities and units should be handled differently.

Digital support is overdue

"Computer programming is the art of telling another human being what one wants the computer to do"

– Donald Knuth

- We have had support for maths for decades (1970s), but not for quantities
- Attempts so far seem to have underestimated, or completely misunderstood, metrology concepts
- The solution is to recognise tacit knowledge and capture it explicitly in a digital supporting layer

Digital support for quantities and units of measurement has remained elusive:

we have had support for mathematical computation for more than half a century but there is nothing yet for quantities and units.

Why? What makes this so hard?

One difficulty is the tacit knowledge people use to interpret written information. This needs to be captured explicitly for digital systems.

Another problem is that the concepts associated with quantities and measurement units are not well understood.

This talk will discuss some of the pitfalls and misconceptions that have prevented progress so far.

And we will see that some change is needed.

Our proposal is to use digitalisation as an opportunity to make those changes in a way that will not be disruptive.

Measurement Standards Laboratory of New Zealand

Money, money, money, …

Accounting uses **numbers** (single currency unit)

Names, like ‘dollar’, label the quantity: as being money

Or they represent multiples or submultiples (dollars and cents)

Non-decimal currencies require special arithmetic
(1 pound = 4 crowns = 20 shillings = 240 pence = 960 farthings)

Tacit knowledge is sometimes used in context
($1 in New Zealand is not the same as $1 the USA)

Having no money is the same in all currencies (a common zero)

A single factor can convert from one currency to another, e.g., 1 NZD = 0.70 USD

Wikimedia commons

To get started, let’s consider a non-physical quantity: money.

Numbers alone will be used to actually account for money (computation requires everything to be converted to a single currency): company balance sheet, personal finances, etc.

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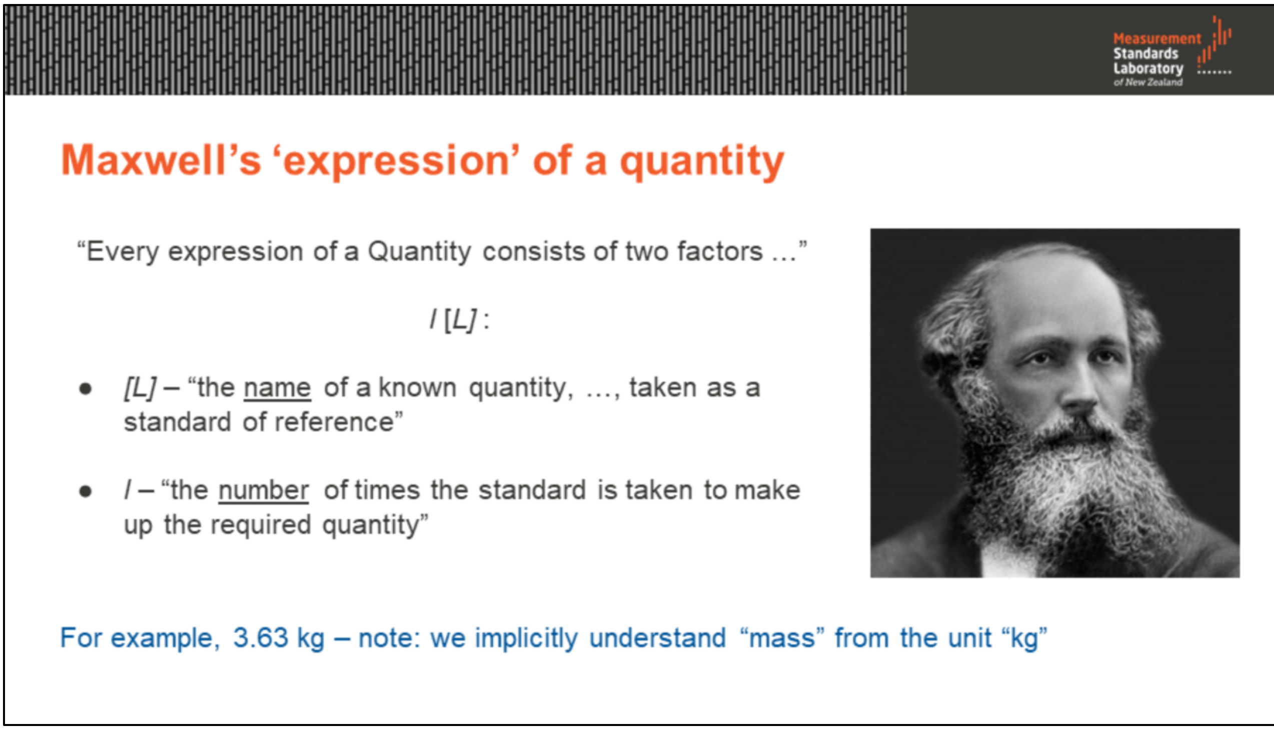

Banks have developed digital representations for money, so why are quantities still a problem?

Well, digital systems must identify the kind of quantity as well – which is not a problem in Finance.

In the 19th century Fourier and Maxwell introduced the ideas we now use to express measurement units.

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Note, Maxwell explicitly assumed that kinds of quantity could be inferred from units. But that is not always so in the SI.

Unit-independent (quantity) equations!!

Unit-independent expressions implicitly contain the two factors.

$$v = \frac{l}{t} = \frac{l[L]}{t[T]}$$

$$= \frac{l}{t}\,\frac{[L]}{[T]} = v[V]$$

The resulting factors may be calculated separately

$v = l/t$ Value

$[V] = [LT^{-1}]$ DIMENSIONS!!

The ideas of Maxwell and Fourier intended to facilitate the expression of physical relationships; they support unit-independent equations for physical quantities.

(There are two interpretations of such equations: numeric and symbolic. For digitalisation, we focus on the numeric.)

Numerical interpretation requires involves the two factors.

Here is the example of speed, a quantity that depends on displacement and duration.

One of the equations is the familiar dimensional expression for speed: dimensions are inherently part of a Maxwellian expression of unit-independent equations.

The SI constructs derived units and unit-names from dimensional expressions (here, metres-per-second).

This was a brilliant innovation. But today there is confusion about how it actually works.

The unit is ultimately a number

"The formulae at which we arrive must be such that a person of any nation, by substituting for the different symbols **the numerical value of the quantities as measured by his own national units**, would arrive at a true result."

-- *J C Maxwell*

reference	unit	context	value
M	lb	$\overset{\text{Imperial}}{=}$	1.0
		$\overset{\text{metric}}{=}$	0.454
	kg	$\overset{\text{Imperial}}{=}$	2.20
		$\overset{\text{metric}}{=}$	1.0

So, it is worth considering how the Maxwellian system is intended to work.

Maxwell sought a numerical result (remember that accounting needs numbers): the unit symbol is <u>the value of the unit reference when measured in the local measurement system</u>.

There are two steps involved:

1. the name of a unit is specified for *M;* this is the name of a conversion factor variable
2. a value is then assigned by the consumer, who needs to know the initial and final units

Note we now see why ordinary rules for arithmetic apply: everything is real-valued; when we divide length by time we are manipulating scale factors, which are just numbers!

We can think of a dimension as a class of similar scales for a quantity (like all currencies for money).

Dimensions and units are really about scale

- Different quantities may have the same dimensions and units
  - Heat capacity & entropy (J/K)
  - Electric current & magnetomotive force (A)
  - Torque & work (N m)
- Dimensionless quantity units are indistinguishable!
  - Aspect ratio, emissivity, refractive index, etc
  - All have the same generic unit in the SI, called “one”
  - All have the same dimensional expression
  - Special names must be used (e.g., radian)

Terms appearing in quantity equations are often considered quantities, and dimensions are thought to identify quantities.

This does not work in practice

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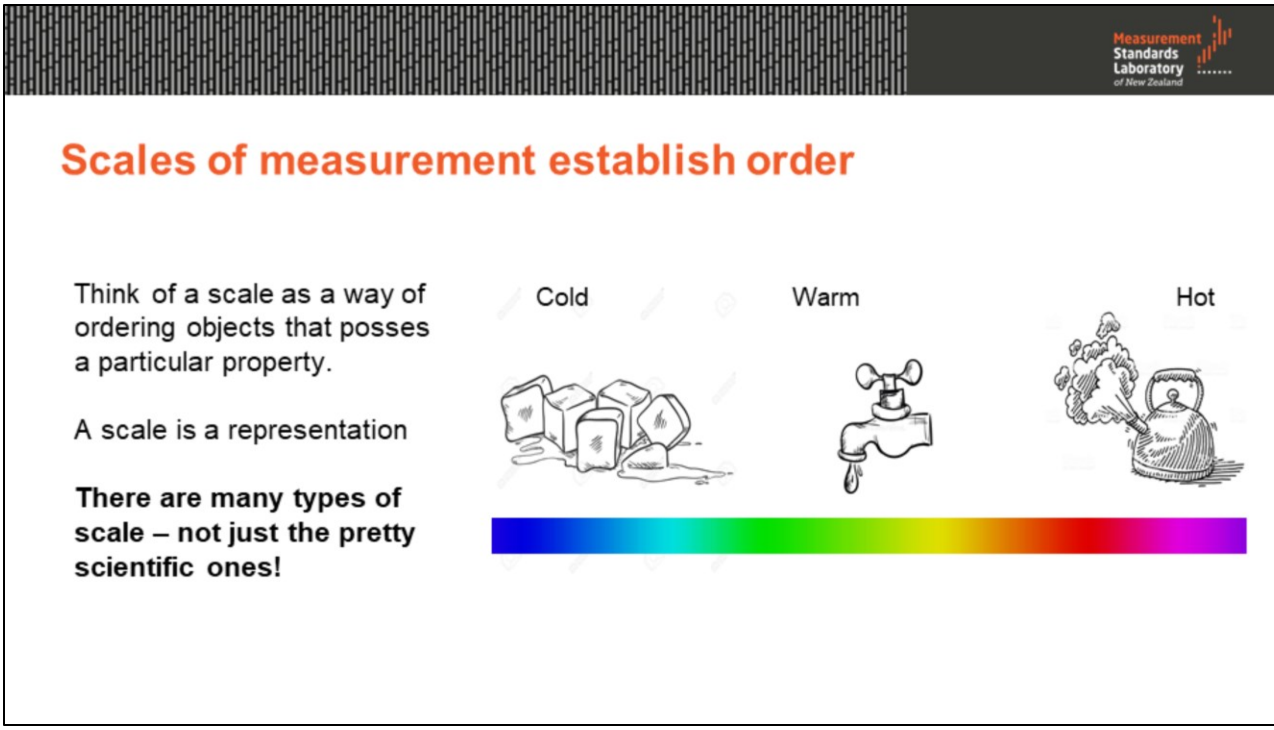
Also, unless special names are assigned to dimensionless units, there is no SI name for them!

For example, the SI defines the “radian” as the name of a dimensionless unit for angle.

Since 2019, the SI has been established by a set of defining constants and equations of physics.

However, **only** because we must provide continuity, our notation still uses the Maxwellian idea of base/derived units. This is a legacy problem.

A modern digital system must not adopt the deprecated framework of base/derived units for internal representations!



The terms in a quantity equation are strictly limited to one type of measurement scale: the ratio scale.

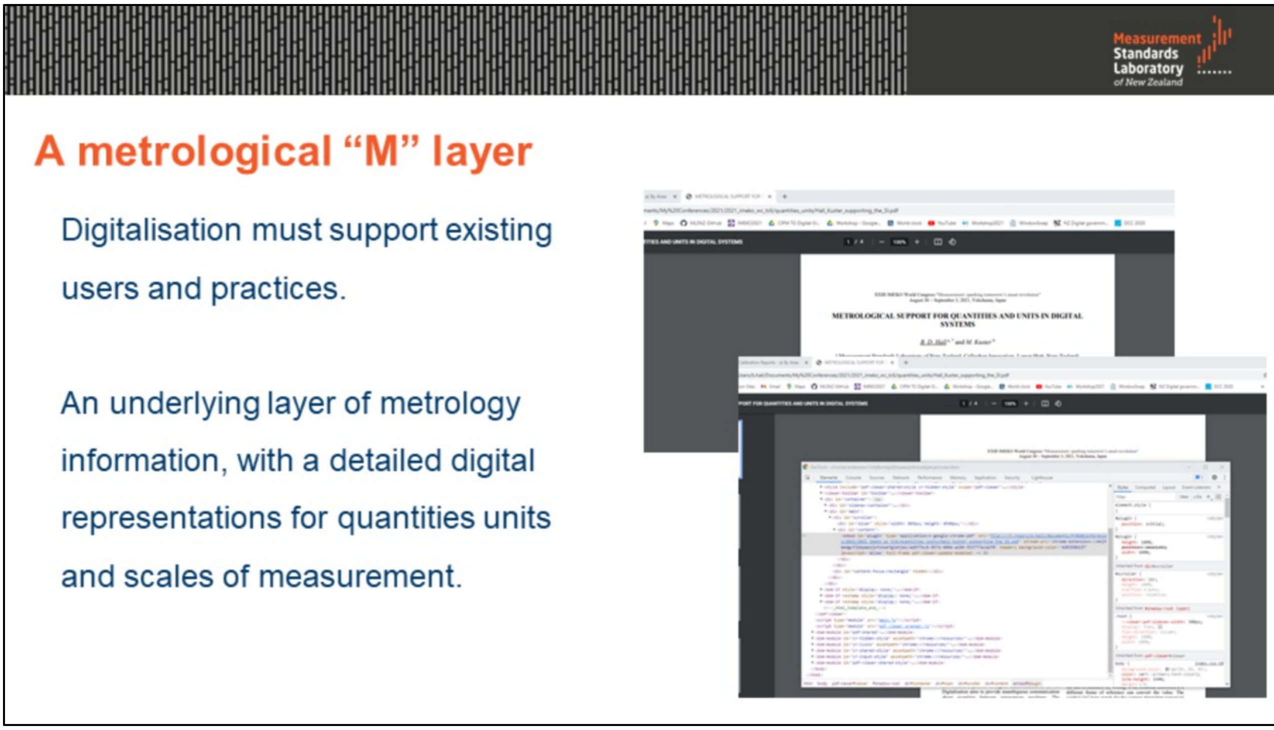
This is the classic scale for physics quantities like length, mass and time (also the case for decimal currencies).

But there are other important types of measurement scale: sometimes the zero is offset (e.g., Celsius and Fahrenheit); sometimes things are merely ordered (Beaufort wind, Scoville scale for chilli peppers, etc).

A more general representation of quantities and units is needed to support various types of measurement.

We should think of the measurement of a quantity as establishing an order among objects possessing that quantity.

Digital infrastructure should accommodate all types of scale in use, including non-decimal numerical scales, like the cyclic behaviour of angles (c.f., non-decimal currencies).



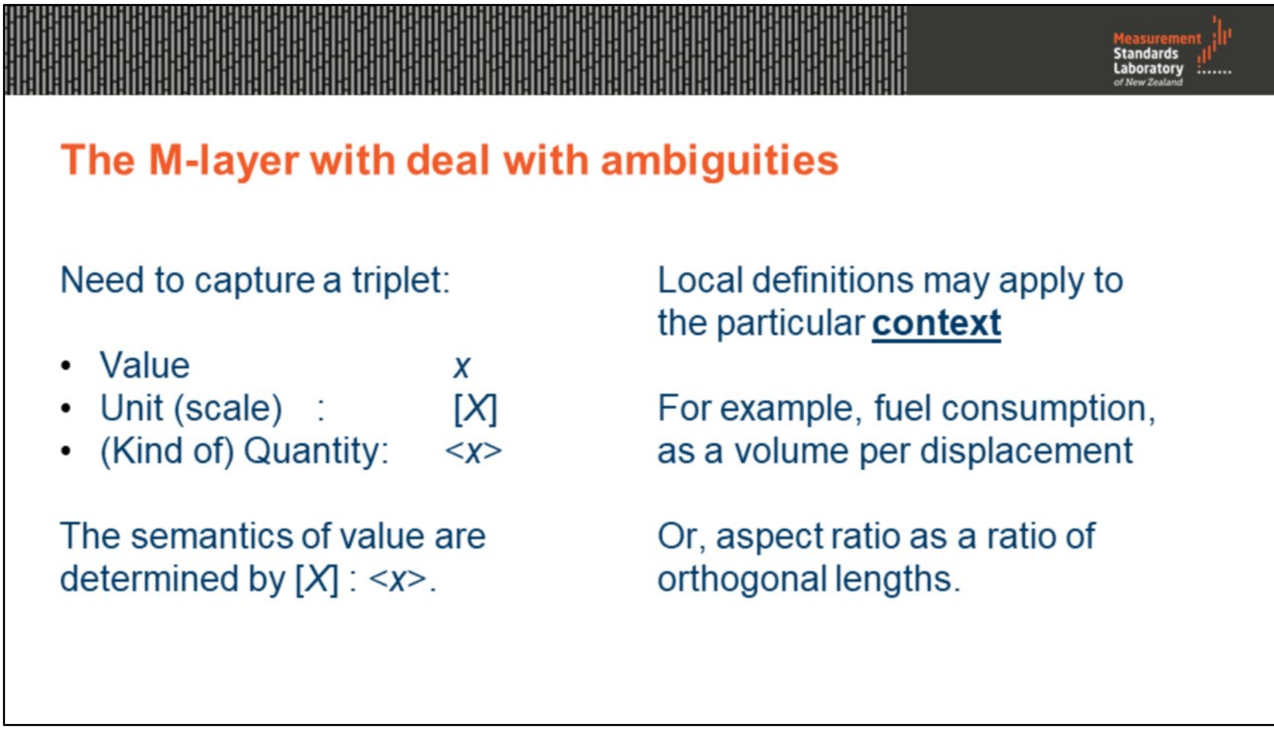
However, most people will not change the way they use units => digitalisation must take care of these details and go beyond the ideas embedded in SI notation.

It is common for digital systems to hide aspects of representation e.g., the rendered form of web page vs the source.

This is quite different from written communication, where all information and complexity is visible to the reader.

We call this metrological layer of support the M-layer.

The M-layer will hold a detailed digital representation of quantities and units, which can be rendered in different ways.



We need three things to interpret a quantity value in context

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The dual nature of the traditional “unit” must be unpacked to identify quantity independently of the unit.

While users specify x [X], we actually need x [X] <x>, where <x> is the kind of quantity.

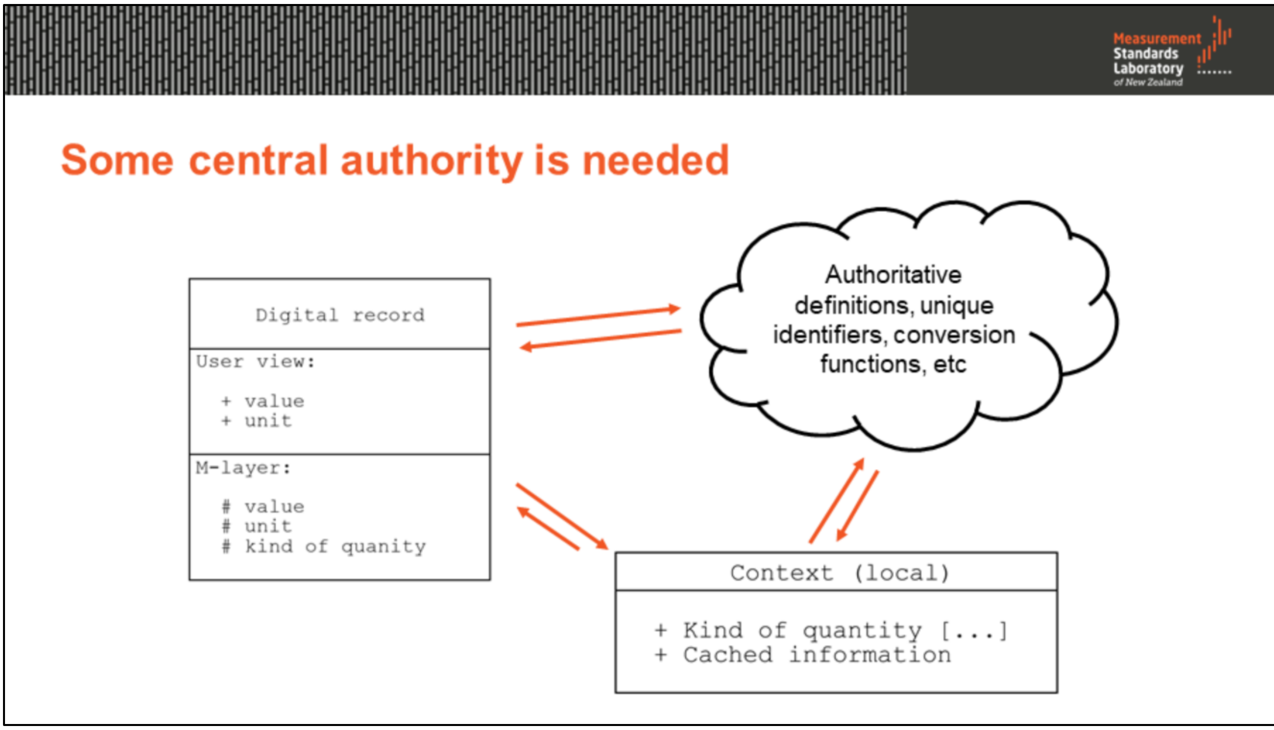
The kind of quantity can usually be elicited from the context.

When the kind of quantity is known, we can avoid existing limitations of SI notation

We can

- Disambiguate quantities that share the same unit
    - Absolute temperature vs ITS-90 temperature

- Convert values from one scale to another



We will need centralised definitions for fundamental quantities and scales.

To ensure interoperability we will need unique identifiers for:

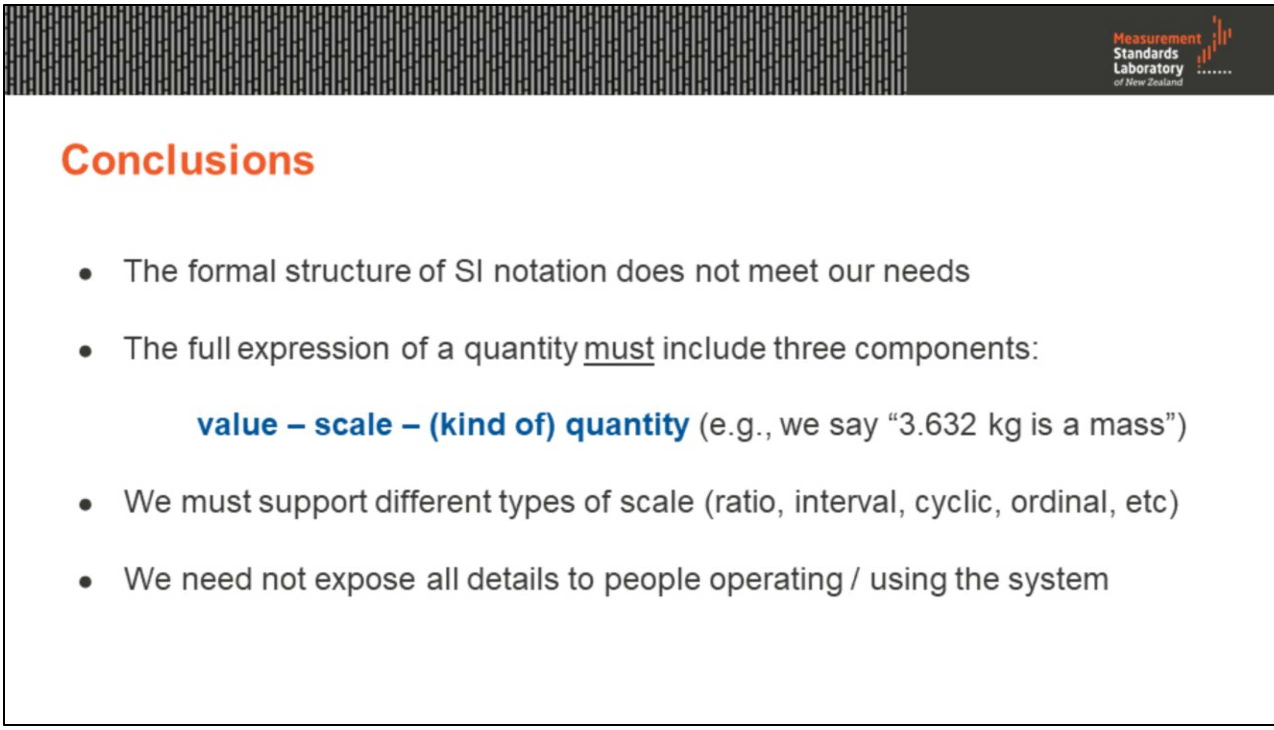
- basic quantities,
- scales of measurement

Conversion between scales depends on both types of scale and is indexed by a pair of scales (initial -> final).

We also envisage local application-specific contextual information

For instance, fuel consumption could be declared a quantity of interest, in terms of fuel volume and distance of travel.

In this way, the local features of a dataset can be described, without requiring every quantity to be defined centrally. Peer-to-peer communication of contextual information would allow M-layer infrastructure to grow gradually, without needing centralisation.



The main points we wish to convey are as follows:

- The formal SI structure is inadequate: the idea of base and derived units should be abandoned,

  but it is possible to extend / generalise support so that other types of measurement can be represented

- An M-layer would provide an unobtrusive way to do this

- Key notions are:
  - Kind of quantity
  - Unit (scale)
  - Numeric properties (of values)
  - Functions to convert between scales

- An M-layer would also be helpful in managing other aspects of digital transformation, such as:
  - Uncertainty/traceability trees
  - Semantics of data processing outcomes


Thank you
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